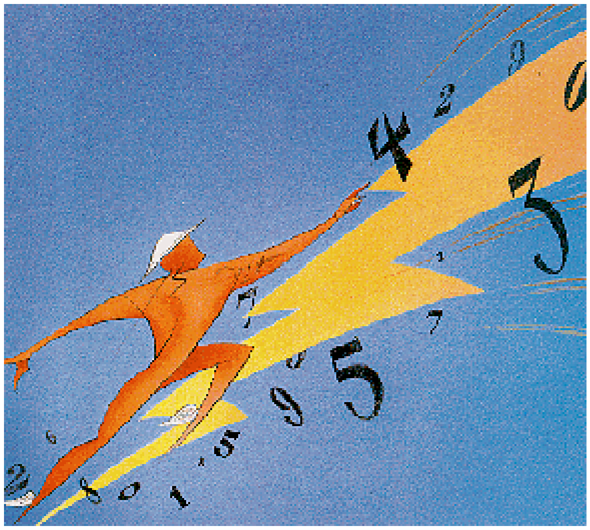
Smoothing Data With Less Lag

by Patrick G. Mulloy



Last time, first-time STOCKS & COMMODITIES contributor Patrick Mulloy discussed basic moving averages, introduced a new filter called DEMA1 and demonstrated a method with which to utilize exponential moving averages. Mulloy also explained how this new filter could be used in the moving average convergence/divergence (MACD) indicator. Now, Mulloy summarizes with more filtering techniques for the MACD and trading the NASDAQ.

Two opposing properties are always at work in the standard moving average smoothing used in technical analysis, increasing the moving average (MA) length to cull more random fluctuations but in so doing thereby increasing the lag between the MA and the data. In my first article, I discussed the industry standards, the simple moving average (SMA, which is a straightforward moving mean of the data) and the faster-responding exponential moving average (EMA). Both the SMA and the EMA smoothing indicators have the same lag in the steady state long term, which is:

$$(1 - \alpha)/\alpha$$

where $\alpha = 2/(w+1)$ and *w* is the moving average period. In terms of the MA period *w*, the lag is:

$$(w-1)/2$$

A new exponential moving average called DEMA1 was introduced in the previous article, which eliminated this lag for the steady state (see sidebar "Calculating TEMA1 and DEMA1").

Now, here are two other moving averages. One is known as TEMA1, which is an extension of the multiple smoothing technique using single, double and triple EMAs, while the other is a subset of a modification of the double-smoothing technique to include a second smoothing constant to smooth the trend of the data model, which I call DEMA2.

TEMA1

When the time series to be modeled is more complicated than a simple linear trend (curvilinear), multiple smoothing leads to the next progression in the polynomial model — the quadratic regression equation:

$$Y_t = \beta_0 + \beta_1 t + ½\beta_2 t^2 + \varepsilon_t$$

where the β (beta) parameters are the regression coefficients and ε (epsilon) is the random error in the data series. For example, in Figure 1 the weekly close of NASDAQ for the 26-week period beginning August 14, 1987, and ending February 5, 1988, is presented. The plot of the closing price is more complicated than a straight-line linear regression, and using a quadratic regression equation is an improvement over the linear regression to model the closing price. Consequently, using a quadratic regression equation involves using three regression coefficients, which leads to triple smoothing techniques.

Time series forecasting derives the estimates for the three model regression coefficients in a fashion similar to the linear model (DEMA1). As we are not forecasting, we are only concerned with the estimate for the nontime (*t*) related coefficient β_0, which involves three EMAs. The EMAs are as follows:

1 $EMA1_m = \alpha C_m + (1-\alpha)EMA1_{m-1}$

2 $EMA2_m = \alpha EMA1_m + (1-\alpha)EMA2_{m-1}$

3 $EMA3_m = \alpha EMA2_m + (1-\alpha)EMA3_{m-1}$

As you can see, the first EMA smoothes the closing price, while the second EMA smoothes the first EMA and the third EMA smoothes the second EMA. This process is known as *triple smoothing* as used in technical indicators such as TRIX, which can be found in technical analysis software.

Our first indicator is smoothed with a subset TEMA1 of the one-parameter triple exponential moving average. The formula for TEMA1 is:

$$TEMA1 = 3EMA1 - 3EMA2 + EMA3$$

As seen from the previous equations, EMA3 is the standard implementation of what is commonly known as a triple exponential moving average. However, the definition of TEMA1 is not a "simple" triple EMA, but rather a *composite* of a single, double and triple EMAs, which eliminates the lag when there is a trend. For example, the TRIX indicator is a standard triple EMA of the one-unit time percentage rate of change of the price. A faster-responding TRIX can be implemented with DEMA1 or TEMA1. Figure 2 presents a comparison between the one-parameter triple (TEMA1) and the double EMA (DEMA1).

INITIALIZING TEMA1

As three regression coefficients must be estimated, there are three smoothed averages to calculate (EMA1, EMA2 and EMA3). Each of these EMAs requires an initial value for the basic recursive EMA equation. In a manner similiar to the DEMA1, TEMA1 also uses a regression polynomial over a subset of the data to use as values in the initialization formulas, but here it uses a quadratic regression. The TEMA1 initialization formulas are provided below. First, define some basic variables and ranges:

C = Closing price vector

FIGURE 1: *Presented is 26 weeks of closing prices with a quadratic and linear regression of the prices. When the data is complicated, then a quadratic or multiple regression should be considered.*

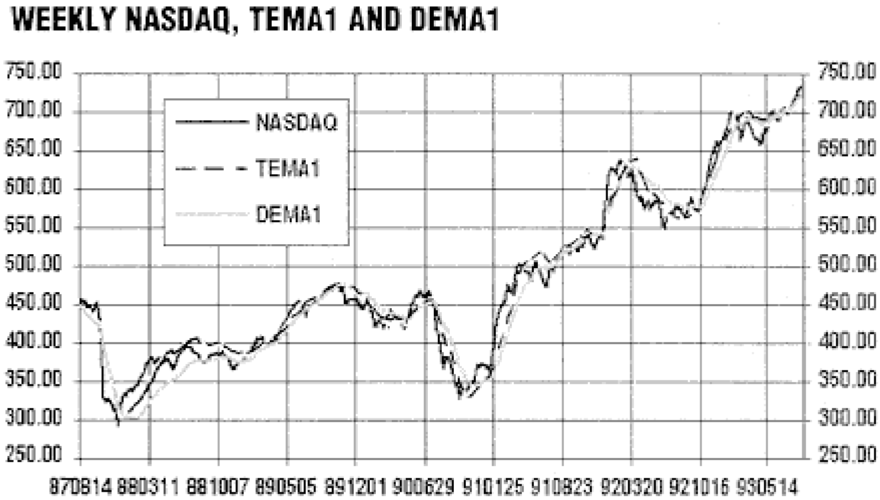


FIGURE 2: *Comparison of 26-week smoothing using TEMA1 and DEMA1 shows that TEMA1 has a closer approximation to the data than DEMA1 does, but it over- and undershoots more at major long-term trend changes.*

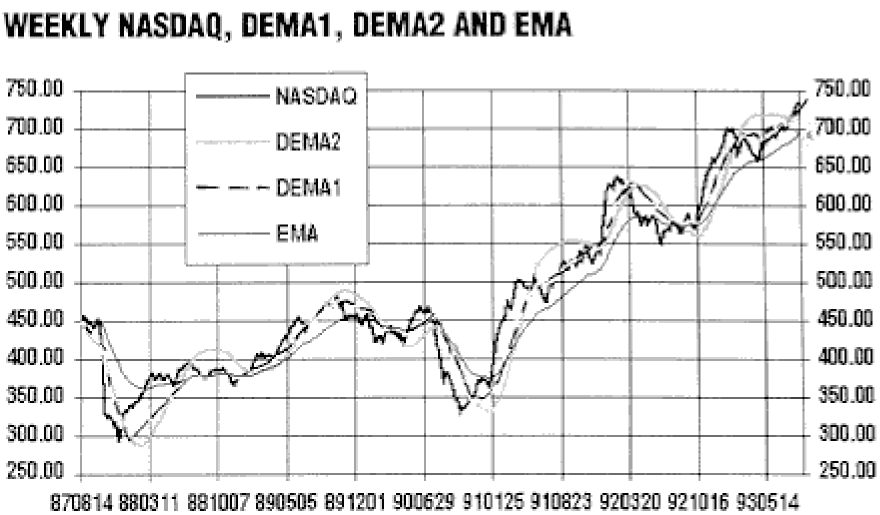


FIGURE 3: *With beta greater than alpha, DEMA2 shows high overshoot at the trend changes.*

$m = 1 .. N - 1$

$N = \text{length}(C) = 316$

$i = 0 .. N - 1$

$t_i = i$

$w = 26$

$k = 0 .. 2w - 1$

$\alpha = 2/(w+1) = 0.074$

Note we are starting at point zero as our starting point and not point 1. Next, estimate regression initial values using the variables for a quadratic regression:

Point 0 to 2w-1

$x_k = t_k$

$y_k = C_k$

Next, calculate regression coefficients using matrix algebra for a least-squares solution for a parabola $y = a_0 + a_1x + a_2x^2$ (for computations not shown, refer to sidebar "Calculating TEMA1 and DEMA1").

$a_0 = 447.753$

$a_1 = -7.876$

$a_2 = 0.146$

The initialization equations for each EMA using the coefficients from the least-squares solution are:

$$EMA1_0 = a_0 - \frac{1-\alpha}{\alpha}a_1 + \frac{(1-\alpha)(2-\alpha)}{2\alpha^2}a_2 = 569.926$$

$$EMA2_0 = a_0 - 2\frac{1-\alpha}{\alpha}a_1 + \frac{2(1-\alpha)(3-2\alpha)}{2\alpha^2}a_2 = 714.912$$

$$EMA3_0 = a_0 - 3\frac{1-\alpha}{\alpha}a_1 + \frac{3(1-\alpha)(4-3\alpha)}{2\alpha^2}a_2 = 882.712$$

Next, calculate EMAs with above initializations:

1 $EMA1_m = \alpha C_m + (1 - \alpha)EMA1_{m-1}$

2 $EMA2_m = \alpha EMA1_m + (1 - \alpha)EMA2_{m-1}$

3 $EMA3_m = \alpha EMA2_m + (1 - \alpha)EMA3_{m-1}$

Then, calculate $TEMA1_0$:

$$TEMA1_0 = 3EMA1 - 3EMA2 + EMA3$$

$$TEMA1_0 = 3(569.926) - 3(714.912) + 882.712$$

$$TEMA1_0 = 447.754$$

See sidebar "Calculating TEMA1 and DEMA1" for a spreadsheet version. Next, we will discuss smoothing with a subset of the Holt-Winters exponential model, also known as the two-parameter double exponential moving average.

DEMA2

This smoothing technique is an expansion of the one-parameter double exponential moving average (DEMA1) in that it explicitly recognizes the secular trend (long term) of the time series but at the same time includes a second smoothing constant specifically for the trend (hence a two-parameter model). The time series can be adequately described by a simple linear regression line:

$$y_t = \beta_0 + \beta_1 t + \varepsilon_t$$

where the beta parameters are slowly changing with time. The Holt-Winters two-parameter double exponential smoothing approach employs two smoothing constants (α and β, that is, two parameters) and two components (*S* and *T*) that are simultaneously smoothed (double smoothing). The first component *S* is the stationary or permanent component and is the estimate for the *y*-intercept β_0 (the component that we are concerned with). The second component *T* is the trend component, which is the estimate for the slope β_1. These two components are separate interrecursive equations that must be calculated simultaneously. Analogous to the single EMA, these two recursive equations require seed values for the iteration process to begin.

Although some latitude exists in choosing the initial values, the simple choice of the value of the time series at time zero is not the best. It is better to calculate the linear regression coefficients at the beginning of the series for a selected period (typically the same as the MA period *w*). Unfortunately, because of the recursive nature of the component equations, the Holt-Winters smoothing algorithm cannot be implemented in programs such as MetaStock. It can, however, be implemented in the following paragraphs with a mathematical program called Mathcad or in the Excel spreadsheet program.

To calculate DEMA2, the first step is to derive the seed values. The window range will be:

$$k2 = 0 .. w - 1$$

The variables for the linear regression are $x2_{k2} = t_{k2}$, $y2_{k2} = C_{k2}$ (C_i is a vector containing all the closing prices). Next, calculate the seed values using a linear regression:

S_0 = intercept(x2,y2) = 448.352 and

T_0 = slope(x2,y2) = -6.162

The two smoothing constants used in DEMA2, α and β, are user-selected depending on the amount of smoothing desired. Alpha is the same smoothing constant as defined earlier for a simple EMA, while beta is the smoothing constant for the trend portion. Both α and β are between zero and 1. In this case, β was

BUY/SELL SIGNALS USING THE MACD ON THE WEEKLY NASDAQ

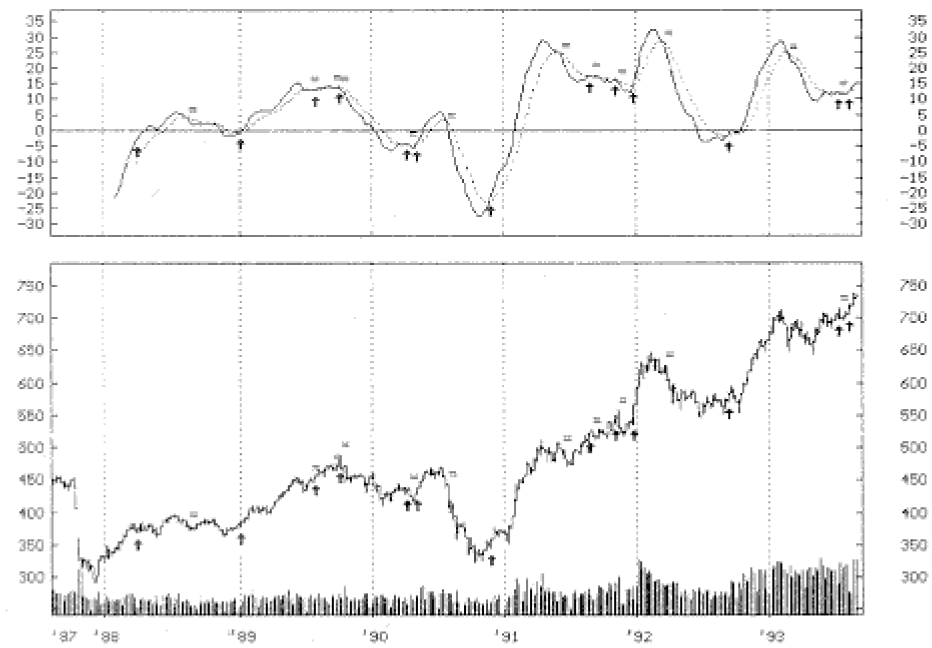


FIGURE 4: *The MACD has 12 buy signals starting from October 1988, with six whipsaws.*

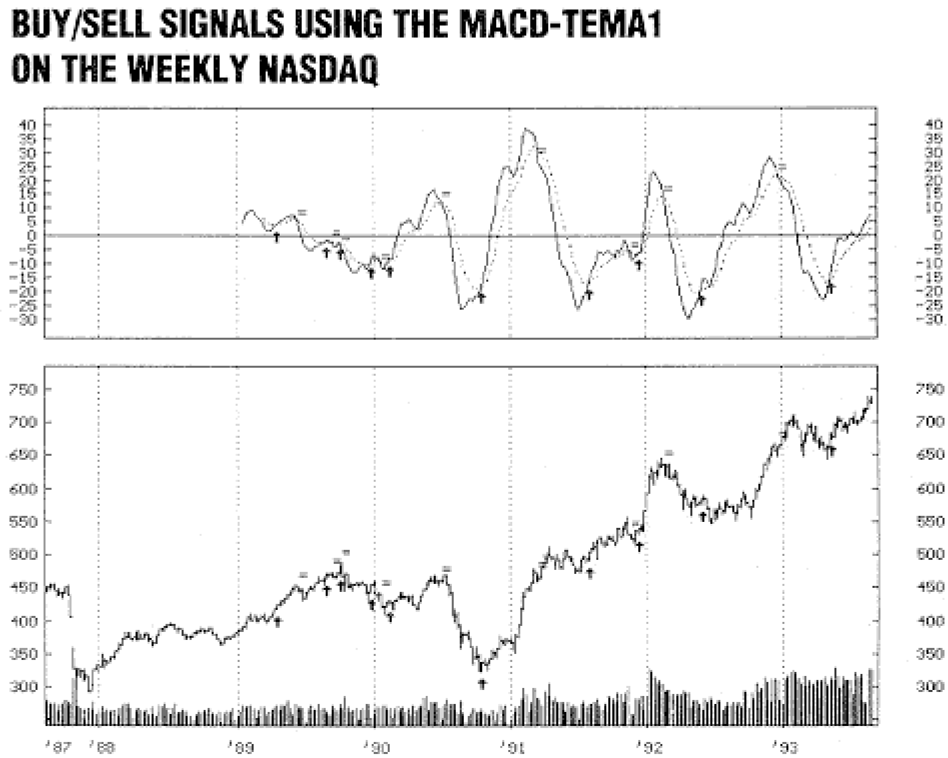


FIGURE 5: *The MACD-TEMA1 (12, 26, 9) has 11 round trips, just like the standard MACD, but with 50% more profit and two fewer drawdowns. Although not visible on the MetaStock plot of the MACD-TEMA1, the two preceding buy/sell signals were on 12/9/88, 3/17/89 and 9/30/88, 11/11/88 (available through Mathcad analysis). These trades are included in the analysis to keep the timeframe for all MACD comparisons the same (October 1988 to August 1993).*

chosen as a factor of α by means of the period *w*, but it does not have to be done that way. Thus, our constants are:

$w = 26$

$\alpha = 0.074$

Beta period is $bw = (0.5)(26) = 13$

$\beta = 2/(bw + 1) = 0.143$

The two components *S* and *T* are defined as

$$S_m = \alpha C_m + (1 - \alpha)(S_{m-1} + T_{m-1})$$

$$T_m = \beta(S_m - S_{m-1}) + (1 - \beta)T_{m-1}$$

The stationary component (S_m) is nothing more than an EMA of the time series data with the addition of a weighted adjustment for the previous smoothed trend value to provide for the growth or decay in the trend. From the last equation, one can see that the trend component *T* is a weighted average of the most recent change in the smoothed stationary value ($S_m - S_{m-1}$) and the trend estimate of the previous time component (T_{m-1}).

Through some simple algebraic manipulation, the above equation for T_m can be shown to be equivalent to:

$$T_m = \alpha\beta[C_m - (S_{m-1} + T_{m-1})] + T_{m-1}$$

The reason to do this is to get rid of S_m so that the two interrecursive equations can be formulated into an array for simultaneous calculation on a per-point basis. Before S_2 can be calculated, T_1 must be calculated and stored in memory and so on. Then the equations can be rewritten and expressed as an array:

$$\begin{pmatrix} S_m \\ T_m \end{pmatrix} = \begin{bmatrix} \alpha C_m + (1-\alpha)(S_{m-1} + T_{m-1}) \\ \alpha\beta\left[C_m - (S_{m-1} + T_{m-1})\right] + T_{m-1} \end{bmatrix}$$

In this case, the final moving average of interest is S_m. For sake of name recognition, the final MA will be redefined as DEMA2, which is S_m (see sidebar “Calculating DEMA2 and the MACD”), and Figure 3 displays a comparison of DEMA2, DEMA1 and an EMA:

DEMA2 = S

MACD IMPLEMENTATION OF NEW MAS

Now that the two new moving averages have been defined, let us implement them into one of the standard technical indicators, MACD (moving average convergence/divergence). The standard MACD is the difference between a 12-unit EMA and a 26-unit EMA with a nine-unit EMA of the difference superimposed as a trigger level for buy/sell signals. Figures 4 and 5 compare a standard MACD with a

TEMA1 implementation of the MACD. The arrows and equal signs show the points at which buy/sell signals occur from the crossovers by the nine-week EMA of the MACD.

Starting from October 1988, the faster-responding MACD-TEMA1 had the same number of buy signals (12) as the standard MACD but more than a 50% increase in total profit. The profit analysis was run using MetaStock's System Tester and algorithms that I personally developed in Mathcad. The MACD-TEMA1 had 11 round-trip trades with four losses, while MACD had 11 trades with six losses.

Comparing the MACDs for TEMA1 with DEMA1 from my previous article, the one-parameter double EMA fared better than the one-parameter triple EMA. The DEMA1 MACD had two fewer trades, approximately a 20% improvement in overall profit, and one fewer drawdown. Experimenting with the MA periods for the MACD-TEMA1 showed that the 16, 39 pair was better than the 12, 26 pair and had eight round-trip trades with one drawdown but more than a 50% improvement in overall profit.

As mentioned previously, the DEMA2 indicator cannot be implemented in MetaStock, so instead it was calculated, plotted and system-tested (profit/loss per trade) with the program Mathcad. Figure 6 is a plot (done in Excel) of the weekly closing prices of NASDAQ along with the MACD-DEMA2, and it is marked with the corresponding buy/sell signals (buy = up arrow, sell = down arrow). For this plot, the initialization formulas were used for each MA — that is, 12 and 26. The MA periods used for the b smoothing constant were 6 and 13. The MACD-DEMA2 had five round-trip trades (starting with the October 1988 buy signal) with one drawdown in mid-1990 of 2.7% before commissions.

The total profit (including closing the open position) of the MACD-DEMA2 was 170% better than the standard MACD, 43% better than the MACD-DEMA1 and 74% better than the MACD-TEMA1. No optimization was conducted for the a MAs for the MACD-DEMA2. MA periods other than 12 and 26 would undoubtedly improve the results for this particular security index. The β MAs were optimized for this test and found to be close to the 1:2 ratio used in this model; however, the same ratio was used for both the 12 and 26 DEMA2. Further testing could possibly show an improvement if different ratios were used for the two DEMA2 indicators prior to subtracting to create the MACD.

Figure 7 provides a quick-look comparison among the MACDs tested in both this article and the first:

Index: Weekly NASDAQ Composite
Commission: 1% per transaction
Profit per $1,000 of initial investment
MACD Periods: 12, 26, 9
Period of comparison: 9/30/88 to 8/27/93

Indicator	Round-trip trades	Losses profit	Total
STD	12	6	441
TEMA1	11	4	689
DEMA1	9	3	838
DEMA2	5	1	1,196

INITIALIZATION TRADEOFFS

Potential users of TEMA1 need not be intimidated by the complexity of the initialization process as

BUY/SELL SIGNALS USING THE MACD-DEMA2 ON THE WEEKLY NASDAQ

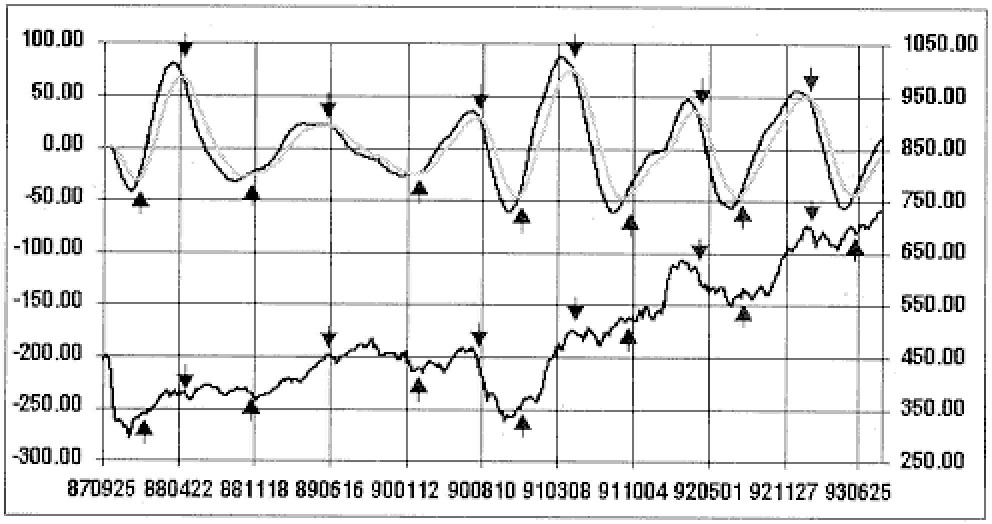


FIGURE 6: *The DEMA2 alpha MAs used are the standard 12 and 26 and the beta MAs are 6 and 13. This MACD shows itself as the best of the four with regard to total profit and only one drawdown in mid-1990. The up arrows are buy signals and the down arrows are the sell signals. This indicator, on its own, eliminated the four whipsaws that occurred in the 1989 and 1991 periods, as opposed to the standard MACD.*

presented here (it requires matrix algebra to solve for the coefficients of a quadratic equation). As demonstrated in my previous article, after approximately two w periods, the DEMA1 (or the TEMA1) indicator(s) will be essentially equal regardless of which initialization period is chosen. If the database of price is at least twice as long as your longest MA period (*w*), then the indicator value using the simple initial value of the database will be more than adequate. Typically, the database length will far exceed the longest w period and a current signal is the object of the search, so there will no detrimental effect by using the simpler initialization procedure.

As far as the DEMA2 indicator is concerned, since it cannot be implemented in MetaStock and another program must be used in any case, it makes the most sense to use the proper initialization of the linear regression. But if for some reason that is not possible, the same generalization holds as for the other two indicators. Use the database initial value at time 0 for S_0 and zero (0) for T_0. If you can approximate the initial trend value, then supply that value for T_0 instead of zero for a faster initial response.

CONCLUSION

Together, this article and the previous one have shown that some definite improvements can be gained in modifying the standard single exponential moving average (EMA). Time series forecasting has for years been using variations on the standard EMA to smooth data and generate forecasting equations. By simply using a subset of the forecasting equations, we have been able to build three new moving averages. The subset used is nothing more than the estimate for the nontime-related coefficient β_0 in the formal exponential moving average techniques used in time series forecasting.

These new MAs have a much faster response during fluctuations than the standard single EMA does and as was shown in the previous article, the lag during steady-state long-term trends can be eliminated. All three of the new MACD formulations acting alone performed better overall compared with the standard MACD. Also for practical applications, the more accurate but more complicated initialization formulas (especially for the TEMA1) can be obviated and replaced with the simple use of the initial database value at time zero.

The utility of the three new MAs presented is only limited by the technical investor's imagination (and time). My analysis with just the MACD implementation is not yet done; indeed, I have not even begun to investigate other indicator implementations such as trend channels.

Patrick Mulloy is an engineer whose current work involves instrumentation data analysis utilizing digital signal processing, regression modeling, statistical hypothesis testing and spectral data analysis. He uses technical analysis in personal investments.

ADDITIONAL READING

Bowerman, B.L., and R.T. O'Connell [1993]. *Forecasting and Time Series: An Applied Approach*, third edition, Duxbury Press.

Brown, Robert Goodel [1963]. *Smoothing, Forecasting and Prediction of Discrete Time Series*, Prentice-Hall.

Gross, C.W., and J.E. Sohl [1989]. "Improving smoothing models with an enhanced initialization scheme," *Journal of Business Forecasting*, Spring.

Mathcad. MathSoft Inc., 201 Broadway, Cambridge, MA 02139, 800 374-6075.

Mendenhall, W., and T. Sincich [1989]. *A Second Course in Business Statistics: Regression Analysis*, third edition, Dellen Publishing.

Mulloy, Patrick G. [1994]. "Smoothing data with faster moving averages," *Technical Analysis of* STOCKS & COMMODITIES, Volume 12: January.

Tuma, Jan J. [1989]. *Handbook of Numerical Calculations in Engineering*, McGraw-Hill.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