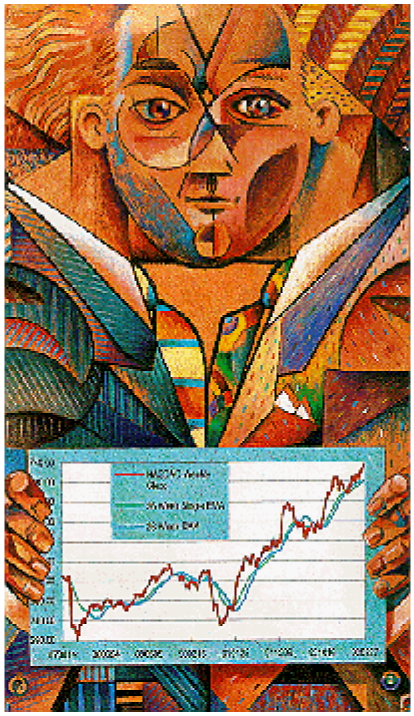

SERIES HH
200

Smoothing Data With Faster Moving Averages

by Patrick G. Mulloy



Has the lag time of moving averages ever irritated you? Well, there is a way around it: a modified statistical version of exponential smoothing with less lag time than the standard exponential moving average that is used in securities technical analysis, a double exponential moving average. First-time STOCKS & COMMODITIES contributor Patrick Mulloy explains.

Moving averages have a detrimental lag time that increases as the moving average length increases. The solution is a modified version of exponential smoothing with less leg time

All moving averages smooth or reduce the noise level of a time series such as closing stock market prices by increasing the moving average (MA) length. But moving averages have an inherent detrimental lag time that increases as the MA length increases. The solution is a modified statistical version of exponential smoothing with less lag time than the standard exponential moving average (EMA) that is commonly used in securities technical analysis. Implementing this faster version of the EMA in indicators such as the moving average convergence/divergence (MACD), Bollinger bands or TRIX can provide different buy/sell signals that are ahead (that is, lead) and respond faster than those provided by the single EMA. In Figure 1, the MACD indicator is applied to the weekly closing price of the NASDAQ composite index. Using the standard MACD EMA lengths of 12, 26 and nine, the indicator generates 11 buy signals with six losses. Figure 2 uses the same filter lengths of 12, 26 and nine, but the filters are not EMAs but are derivations of one-parameter double exponential moving averages (DEMA1). This time, the indicator generated nine trades, with only three losses due to the increased response of the DEMA1 filter. Here are

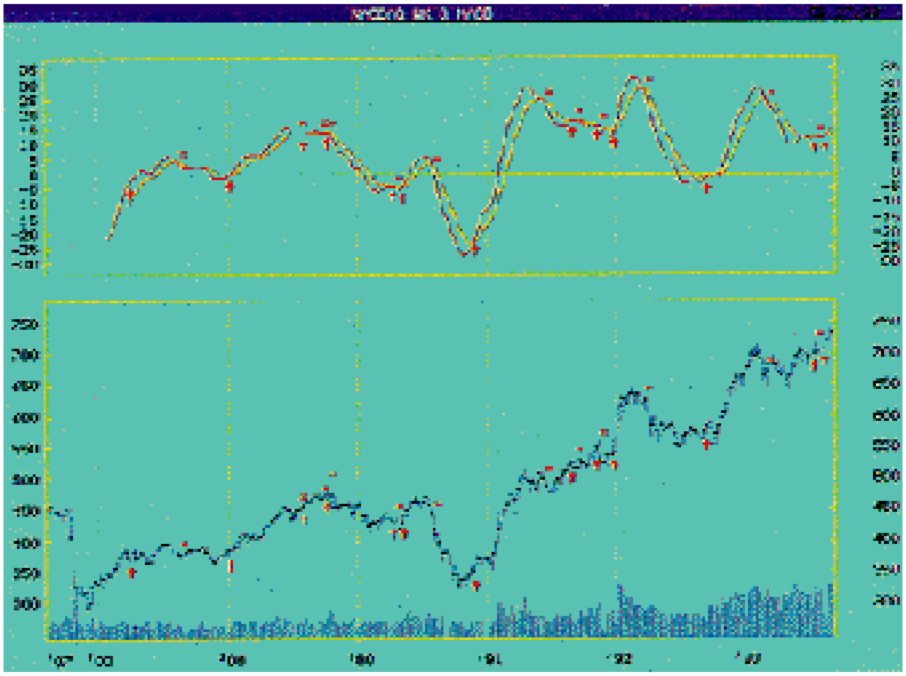


FIGURE 1: WEEKLY NASDAQ AND MACD. *The MACD has 12 buy signals beginning in October 1988 with six whipsaws. The latest buy signal is still open.*

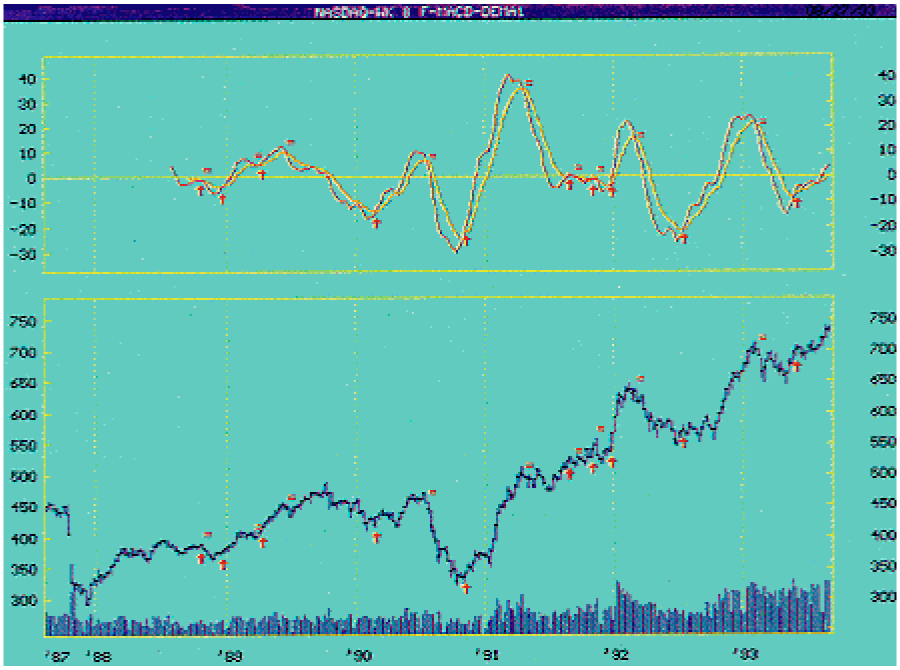


FIGURE 2: Weekly NASDAQ and MACD-DEMA1. *The MACD-DEMA1 (12,26,9) has fewer round-trips than the standard MACD and has almost twice the profit.*

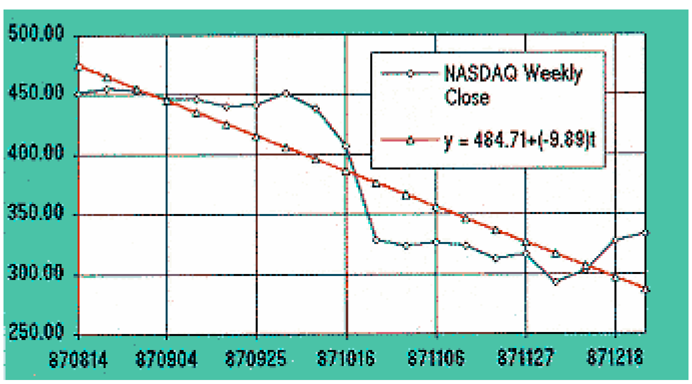


Figure 3: Weekly NASDAQ and Linear Regression. *Calculating a linear regression of the data, we see that* β_0 *(y-intercept) is 484.71 and* β_1 *(slope) is -9.89.*

the attributes of the DEMA1 filters and the methods by which to calculate the filters.

THE BASICS

The term *statistical* is used qualitatively here because exponential smoothing is not based on any formal statistical theory, and for that reason, these smoothing techniques are best regarded as descriptive rather than inferential in statistical terminology. With that in mind, data smoothing by using moving averages is a common methodology in the statistical world of time series forecasting. The moving average smoothing technique removes the rapid fluctuations in the time series so that the secular (that is, long-term) trend is more apparent. Exponential smoothing was originally developed to primarily forecast time series that can be represented by a polynomial function of time. Therefore, in the following descriptions it is assumed that the data is being modeled with the most appropriate regression polynomial of the form:

$$y_t = \beta_0 + \beta_1 t + 1/2\beta_2 t^2 + \varepsilon_t$$

where the β (beta) parameters are called the regression coefficients and ε (epsilon) is the random error in the data series. In a linear regression model as below, β_0 represents the y-intercept and β_1 represents the slope of the line at time t. For example, in Figure 3 we are modeling 20 weeks' worth of daily closing price data of NASDAQ using a linear regression:

$$y_t = \beta_0 + \beta_1 t$$

The straight line through the data represents the linear regression with the y intercept $\beta_0 = 484.71$ and the slope $\beta_1 = -9.89$. In Figure 4, the linear regression is applied four weeks later and the new betas are $\beta_0 = 430.09$ and $\beta_1 = -6.88$. Note that the two lines have different slopes and different y-intercepts. The betas have changed over the two different time periods. The speed with which these betas change affects the type of smoothing filter we should apply to our data when developing trading indicators.

SMOOTHING WITH THE SMA

As a way of introduction, the first and most obvious smoothing method is the straightforward mean (arithmetic average) of the data over a selected period (window) of time *w*. The simple moving average (SMA) uses an equal weighting technique in that each value in the window is weighted by the same factor 1/*w*. The simple moving average is more effective when the parameters describing the time series are changing *very* slowly over time. It is most effective if the time series is effectively constant with time, that is, $\beta 1$ and $\beta 2$ are equal to zero, and so the time series can be represented by the mean of the entire database. The model reduces to:

$$y_t = \beta_0 + \varepsilon_t$$

where β_0 will be the mean of the data, not the y-intercept.

That the mean of the data over widely separated segments of time may vary slowly makes the moving average an effective tool. All technical analysis software packages provide the simple moving average as a default moving average. In mathematical terms, the SMA can be expressed as:

where:

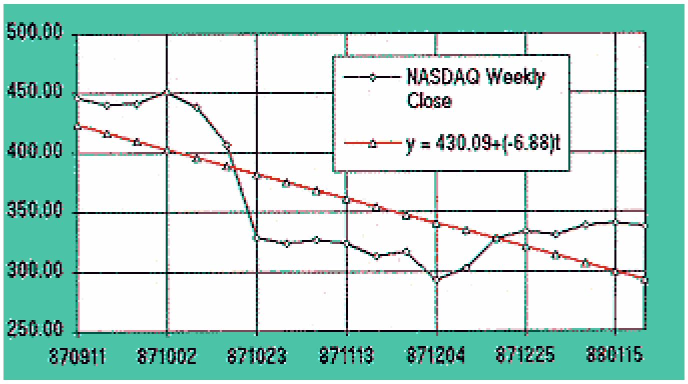


Figure 4: Weekly NASDAQ and Linear Regression. *Moving forward by four weeks and calculating a linear regression of the data, we see that β_0 (y-intercept) is now 430.09 and β_1 (slope) is -6.88. The betas (β) have changed.*

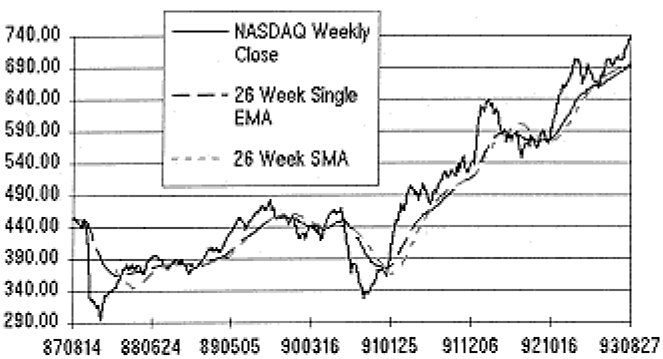


Figure 5: Weekly NASDAQ, 26-Week Single EMA and 26-Week SMA. *The simple moving average and the exponential moving average have similar plots. The exponential moving average is more sensitive to the most recent price action.*

N = Number of records in the time series

C = Closing price vector (range 0 to N - 1)

w = Moving average period

SMA data range: n = w - 1 .. N - 1

Window range: k = 0 .. w - 1

SMOOTHING WITH THE SINGLE EMA

A faster responding average, but not with less steady-state lag, is the single exponential moving average (EMA). The SMA drops off preceding values as it progresses through the time series, which can cause a distortion in the current plot point if a large, abrupt fluctuation occurred in the past. This does not happen with the EMA because its current value always includes a percentage of every preceding point. The smoothing constant or weight alpha (α) in the equation for EMA determines what percent to use of the current data value plus the remaining percentage ($1-\alpha$) to use for the previous smoothed value. The closer a is to 1, the closer the EMA follows the original data — no averaging. With $\alpha = 1$, the EMA will be a duplicate of the original data. In securities analysis, α may be expressed in more meaningful terms as a parameter (*w*) representing the period of time over which the data is averaged.

Exponential smoothing (even more so for the SMA) is most effective when the parameters describing the time series are changing slowly over time. For the single exponential moving average, it is also most effective if the time series is effectively constant with time, that is, β_1 and β_2 equal zero. The model is again:

$$y_t = \beta_0 + \varepsilon_t$$

In reality, the β_0 may be changing slowly over time, but the simple moving average that uses equal weighting may not be the most appropriate smoothing scheme; hence, the exponential moving average that applies unequal weighting is the better choice. More weight is applied to recent observations and thus will respond quicker to a slowly changing trend. Mathematically, the EMA is a recursive equation, meaning that its current value depends on a previously calculated value, as shown below in the definition for EMA:

$$EMA_m=\alpha C_m+(1-\alpha)EMA_{m-1}$$

where:

α = Smoothing constant, $2/(w+1)$

C_m = Closing price period *m*

EMA_{m-1} = Yesterday's EMA

For example, define a data range starting at period 1:

$m = 1 .. N - 1$

Choose MA period, $w = 26$,

Therefore a = 2/(26 +1)

$\alpha = 0.074$

This is our alpha for a 26-period EMA. The value of a, the smoothing constant, is always between zero and 1. Because of the EMA's recursive nature, a *seed* or initializing value must always be chosen for the value of EMA_0.

For the initial value of the EMA, we will substitute the first day's closing price (C_0) for yesterday's EMA calculation:

$$EMA_0 = C_0$$

$$EMA_0 = 451.61$$

We then begin the smoothing process with the second day's closing prices (C_1).

If the EMA formula is factored another way:

$$EMA_m = EMA_{m-1} + \alpha(C_m - EMA_{m-1})$$

This version gives a different insight into the meaning of the EMA. The new smoothed price is equal to the previous smoothed price, plus a percentage a of the difference between the current price and the previously smoothed price.

Typically, in time series forecasting, the smoothing constant a is between 0.01 and 0.30 (*w* = 5.7 to 199). Low values of a (high values of w) mean that the average level of the time series is not changing much over time. If values of a greater than 0.30 appear to be required (for example, greater than a 200-day MA), then you should consider using a different model to represent the data. Figure 5 displays a 26-week moving average using the SMA and the EMA on a plot of the weekly closing price for the NASDAQ index.

SMOOTHING WITH A DEMA1

The first extension from a model represented by the mean of the data:

$$y_t = \beta_0 + \varepsilon_t$$

is a linear trend model

$$y_t = \beta_0 + \beta_1 t + \varepsilon_t$$

and if the beta parameters are changing with time, then the equal weighting of the simple moving average is definitely not an adequate smoothing function. We will see that even the single exponential moving average can be improved upon. From the equation above, it is apparent that now two regression coefficients should be estimated when modeling the market, β_0 and β_1 instead of just β_0 when we look at simple moving averages. This leads to *multiple smoothing*, or in this model case, *double smoothing*.

In time series forecasting literature, the one-parameter double exponential smoothing employs single- and double-smoothed statistics S1 and S2, computed as follows:

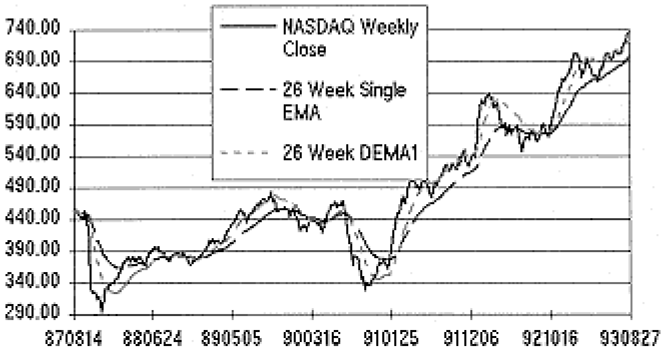


Figure 6: Weekly NASDAQ, 26-Week Single EMA and 26-Week DEMA1. *The DEMA1 shows higher response to the changing prices than the single EMA.*

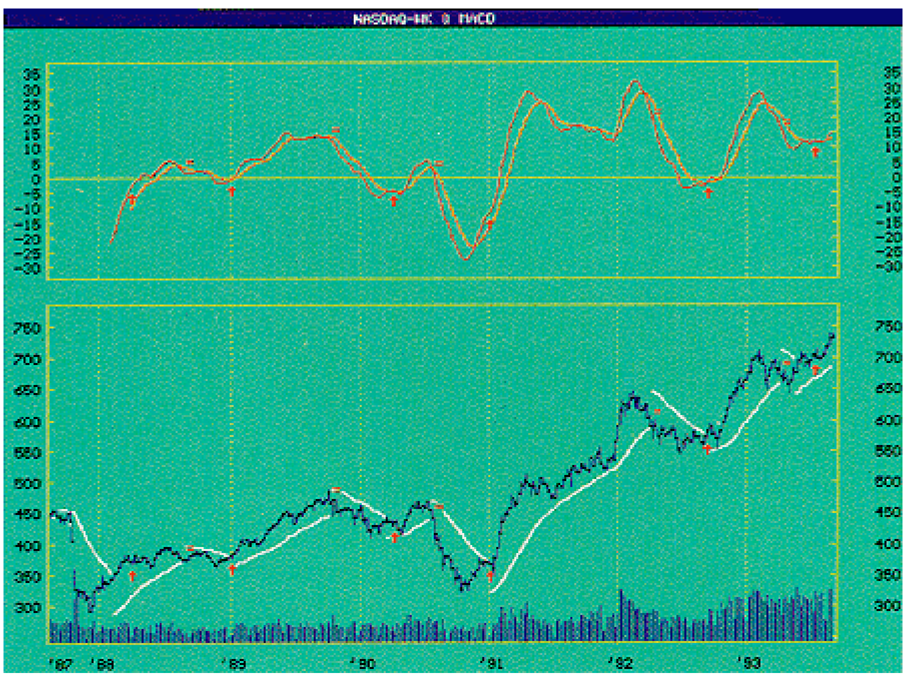


FIGURE 7: Weekly NASDAQ, MACD and Parabolic SAR (0.03/0.06). *The parabolic stop-and-reverse indicator can be used as a trailing stop loss mechanism.*

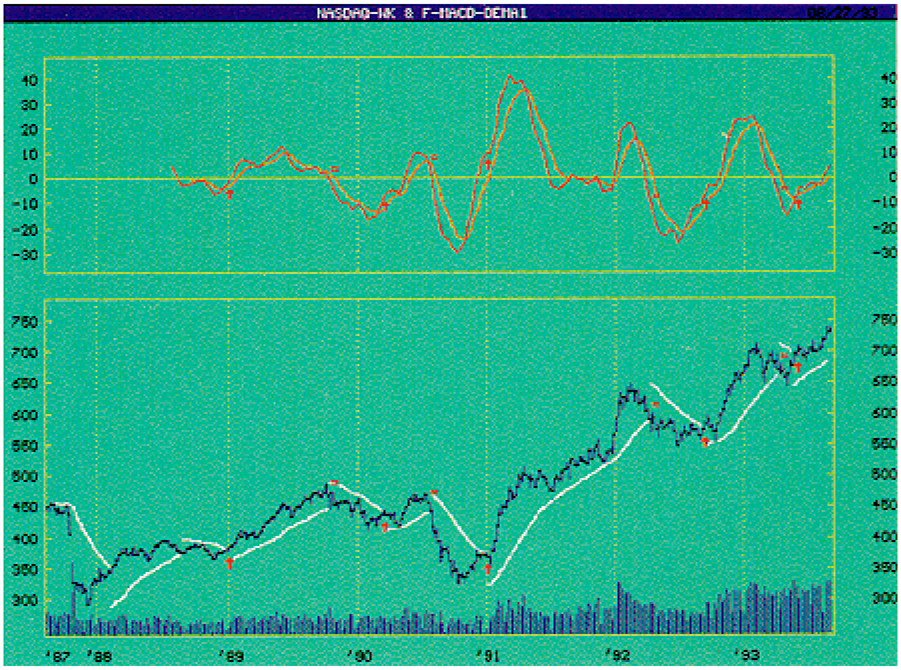


FIGURE 8: Weekly NASDAQ, MACD-DEMA1 and Parabolic SAR (0.03/0.06). *Adding another indicator as a qualifier greatly improved both MACDs, eliminating many whipsaws. MACD-DEMA$_1$ (12,26,9) with stop-and-reverse (SAR) provided dramatically less loss for the correction in August 1990 than the standard MACD with SAR.*

$$S1_m = \alpha Cm + (1 - \alpha)S1_{m-1}$$

$$S2_m = \alpha S1_m + (1 - \alpha)S2_{m-1}$$

S1 is the single exponential moving average of the original data. S2 smoothes S1 exponentially, revealing that it is the double EMA. In formal time series forecasting, however, these two statistics are used to calculate the estimates b_0 and b_1 for the real regression coefficients b_0 and b_1 and then formulate the final forecasting equation for a forecast t time units in the future:

$$y_{t+\tau} = b_0 + b_1\tau$$

The coefficient b1 is used when forecasting at time $t + \tau$ and is not important here. The y-intercept for the trendline at time *t* is b0 and that is what is important. We will estimate the b0 using an improved (less lag-time) moving average (DEMA1), a derivation of the one-parameter double exponential moving average. Before proceeding let us rename S1 and S2 as EMA1 and EMA2 for consistency. The formula for DEMA1 is:

DEMA1 = 2EMA1 - EMA2

Here, we are multiplying the single EMA by two and then subtracting the double EMA. The derivation of DEMA1 is provided in sidebar "DEMA1 derivation." In addition, there is a spreadsheet version presented in sidebar "DEMA1 Excel spreadsheet."

The two components EMA1 and EMA2 each require values at time 0 to initialize the recursive calculations. For best results, the initial values are not the data value at time 0 (C_0). Specific equations have been derived in the literature to provide the initial values. The first thing that must be done is to calculate the coefficients for a linear regression over a subset of the database (typically, the first window period w), which are then used in the initialization equations. The tradeoffs in using the simpler C_0 value versus the initialization equations are discussed elsewhere. The following are steps used to calculate a 26-week DEMA1 using weekly closing prices of the NASDAQ index beginning with August 14, 1987, of 451.61.

Estimate regression initial values:

Given: I = 0 .. N - 1, $t_i = i$, k = 0 .. w - 1

Define variables for a linear regression from point 0 to *w*-1, $x_k = t_k$, $y_k = C_k$

Calculate regression coefficients using built-in functions intercept and slope:

u = Intercept(x,y) = 454.513

v = Slope(x,y) = -6.162

For a 26-week window:

$\alpha = 2/(26+1) = 0.074$

Initialization equations:

$$EMA1_0 = 454.513 - ((1-0.074)/0.074)(-6.162)$$

$EMA1_0 = 531.62$

$EMA2_0 = 454.513 - 2((1-0.074)/0.074)(-6.162)$

$EMA2_0 = 608.72$

Calculate EMA's with above initializations:

$EMA1_m = \alpha C_m + (1 - \alpha)EMA1_{m - 1}$

$EMA1_1 = (0.074)(455.20) + (1- 0.074)(531.62)$

$EMA1_1 = 525.96$

$EMA2_m = \alpha EMA1_m + (1 - \alpha)EMA2_{m - 1}$

$EMA2_1 = (0.074)(525.96) + (1- 0.074)(608.72)$

$EMA2_1 = 602.60$

Calculate DEMA1:

$DEMA1_1 = 2EMA1_1 - EMA2_1$

$DEMA1_1 = (2)(525.96) - 602.60$

$DEMA1_1 = 449.33$

So even though a double exponential moving average (EMA2) is used in the formulation of DEMA1, the one-parameter double exponential smoothing as described here is *not* equivalent to the double EMA itself. Figure 6 displays the dramatic difference between DEMA1 and EMA.

This plot displays the considerable improvement in response by the DEMA1 to the closing price changes versus the standard EMA.

MACD IMPLEMENTATION OF DEMA1

Now that we have defined a new moving average, let us implement it into a standard technical indicator, MACD. The standard MACD is the difference between a 12-unit EMA and a 26-unit EMA with a nine-unit EMA of the difference superimposed as a trigger level for buy/sell signals. See Figures 1 and 2 for a comparison of a standard MACD with a DEMA1 implementation of the MACD. The arrows and equal signs show the points at which buy/sell signals occur from the crossovers by the nine-week EMA of the MACD. Starting from October 1988, the faster-responding MACD-DEMA1 actually has fewer buy signals (10) than the slower MACD (12) with almost twice (+90%) the total profit. The profit analysis was run using MetaStock's System Tester program and Mathcad. The MACD-DEMA1 had nine round-trip trades with three losses, while MACD had 11 trades with six drawdowns.

As all experienced technical investors know, you don't rely on just one indicator to determine your buy/sell signals. If the MACD is one of your trading indicators, you can experiment with a faster-responding MACD and make adjustments as needed to fit your investment philosophy. The simple addition of a parabolic stop-and-reverse (SAR) indicator (using a step value of 0.03 to a maximum of 0.06) as a second qualifier dramatically improves both MACDs (Figures 7 and 8). In particular, the two whipsaw losing trades in the August to November period of 1991 for both MACDs are eliminated. MACD-DEMA1 still provides the better overall performance; it exits one week earlier the correction in August 1990, decreasing the loss for that trade by more than a factor of 7, and it enters the last trade on the chart six weeks earlier than with the standard MACD.

Another obvious caveat in using the DEMA1 is that the MA periods being used may have to be lengthened because of the fast response of the indicator. There is always a tradeoff of choosing a long-enough MA period to swamp out random noise but still capture those meaningful shifts in trend early enough.

CONCLUSION

By simply extracting the estimate for the non-time related coefficient B_0 in the formal one-parameter double exponential moving average used in time series forecasting, it has been shown to be an effective modified EMA with much faster response during fluctuations than the standard single EMA. In addition, the DEMA1 is not just a double EMA with twice the lag time of a single EMA, but rather it is a composite implementation of single and double EMAs producing another EMA with less lag than either of the original two. For general use, the more accurate but more complicated initialization formulas can be obviated and replaced with the simple use of the initial database value at time 0.

Next time, I will present the next extension in multiple smoothing called the one-parameter triple exponential moving average (TEMA1) and a variation of double smoothing that includes a second smoothing constant for the trend — the two-parameter double exponential moving average (DEMA2).

Patrick Mulloy is an engineer whose current work involves instrumentation data analysis utilizing digital signal processing, regression modeling, statistical hypothesis testing and spectral data analysis. He uses technical analysis in his personal investments.

REFERENCES

Blau, William [1991]. “Double-smoothed momenta,” *Technical Analysis of* STOCKS & COMMODITIES, Volume 9: May.

Bowerman, B.L., and R.T. O'Connell [1993]. *Forecasting and Time Series: An Applied Approach*, 3rd edition, Duxbury Press.

Brown, Robert Goodel [1963]. *Smoothing, Forecasting and Prediction of Discrete Time Series*, Prentice-Hall.

Gross, C.W., and J.E. Sohl [1989]. "Improving smoothing models with an enhanced initialization scheme," *Journal of Business Forecasting*, Spring.

Hartle, Thom [1993]. “The parabolic trading system,” *Technical Analysis of* STOCKS & COMMODITIES, Volume 12: November.

Mathcad. MathSoft Inc., 201 Broadway, Cambrdige, MA 02139, 800 374-6075.

Mendenhall, W., and T. Sincich [1989]. *A Second Course in Business Statistics: Regression Analysis*, 3d edition, Dellen Publishing.

MetaStock. Equis International, 3950 South 700 East, suite 100, Salt Lake City, UT 84107, 801 265-8886.

Rothschild, Raymond [1992]. ìUnderstanding exponential moving averages,î *Technical Analysis of* Stocks & Commodities, Volume 10: August.

Tuma, Jan J. [1989]. *Handbook of Numerical Calculators in Engineering*, McGraw-Hill.