The Chaology of Markets (a Multifractal trading model)

*There are tons of books, articles, and views on chaos and fractals none of which has **directly** assisted the trader to trade the markets by chaos theory and fractal geometry or in fact shown that those ideas are in any way different to the prospects of traders than current methods. In this article, we end all of that and outline a **Multifractal** trading methodology that shows why and how a method based those sciences is superior to "Technical" and "Fundamental" Analysis and is intellectually accessible to traders.*

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For the concept of a fractal market structure to be of any use at all, we need to know what a price fractal is, where it comes from, what it looks like and how it behaves in terms of the abstractions of chaos theory and the concreteness of traded markets. Now the link between chaos and fractals is rather simple because chaos is the result of an iterative process and fractals are defined by iterates. But until now if you asked to see what a price fractal looks like you will most probably draw a blank. We found and defined what a price or market fractal is and this means we have articulated the fractal footprint of markets (incontrovertibly).[1] However, in order that the reader gains the import of this, and understands the basis for it, it is important to provide some background understanding of the issues involved. In this article a mathematical background is not assumed.

What exactly is Chaos?

Chaos is a mathematical concept and any trader can understand the mathematics of chaos and how it relates to Stock markets, Forex markets, etc. In fact, it can be intuitive, where a trader trades with a need to understand exactly how price moves in real-time. As such, it is a powerful sense by which to understand market dynamics.

It requires an understanding of a map called the logistic map and how it outputs logistic systems.

$$\text{Logistic Map: } Xn+1 = RXn\,(1 - Xn)$$

By noting the behavior of iterates over different rates of change (systems), and the specific character of their fluctuations (using web diagrams), a set of classifications emerge that specify sequences of iterations by their period-doubling behavior, i.e. the number of iterations needed for some seed x to return to some specified marker or point in range. It is this period-doubling behavior in iterated sequences that leads to chaos.

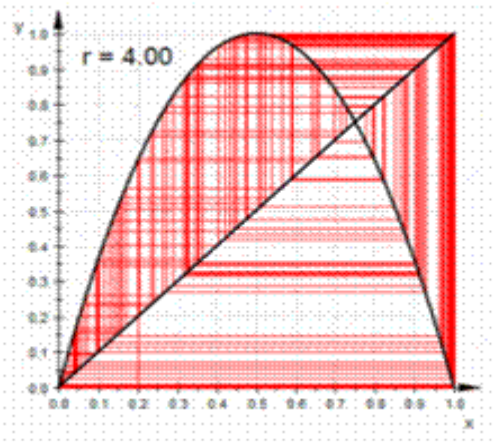

Figure 1: Logistic Chaos: Web Diagram

The underlying operation of the logistic map is one akin to the repeated folding and stretching of the space to which it maps, which leads to exponential divergence in the sequence of iterates. It is by this exponential divergence of sequences that we measure for chaos. So, bifurcation rates explain the relationship between chaos and unpredictability. This is clear if we consider that at high rates of exponential change, small errors multiply at exponential speeds. The dynamics of logistic chaotic systems is summarized by a structural diagram called a bifurcation diagram.

[1] Benoit Mandelbrot in his book "The Mis (Behavior) of Markets," showed how the Iterated Function Systems (IFS) formalism of fractal geometry, may be applied to the market and what a fractal generator is in terms of a model that simulates the market. Our context is different in that we do not speak to a model that simulates or projects the market but one that allows the reading and trading of markets in real-time using **actual** price charts. As such, the need was to find that fractal footprint in situ that defines the overall structure of markets in ways anyone can reasonably conclude to be deterministic of market price.

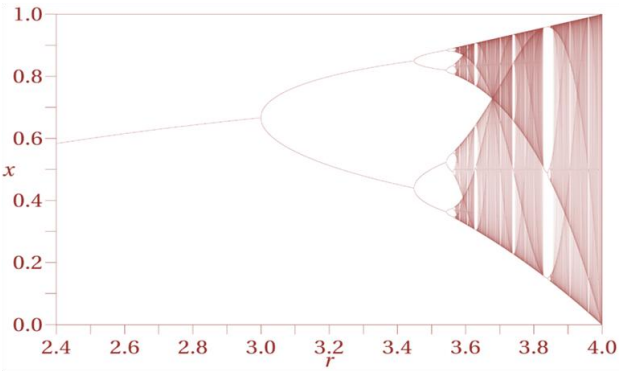

Figure 2: Logistic Bifurcation Diagram

The bifurcation diagram plots the end behaviors of different systems (i.e. the end behaviors of iterated sequences at different intrinsic rates of change) against a measure of change in the dependent variable. It shows the period doubling route to chaos and thus the structure of a chaotic system. This diagram tells us important things that help us better understand market dynamics on a structural level (i.e. given that markets are deterministic, chaotic and nonlinear). One such important piece of information it provides is that chaotic systems are fractal in structure, and so we can equate chaotic behavior to fractal behavior (the bifurcation diagram is self-similar, i.e. fractal). This means that an understanding of fractals and fractal behavior enables measurement and control of the fundamental dynamics of the system. This is important because fractal behavior is less abstract in concept and can be read by the pattern of point displacement in a given system. The chaos game for instance.

Implications of Markets as Chaotic Systems

There are several important (rational, even somewhat moral) implications arising from the knowledge that markets are **nonlinear** and chaotic**.** One crucial implication is that a key premise on which "technical" analysis (TA) is based (history repeats itself) is fallacious. Another key implication is that no linear model of the market is suited to explaining price except in very partial terms (of which both so-called "fundamental" analysis (FA) and "technical" analysis (TA) are examples). This is because the chaotic variable (price in this case) evolves in nonlinear ways. But even more significant is that the absence of nonlinearity in a model describing the market is problematical because chaos needs nonlinearity. **Nonlinearity** is really what helps to make a chaotic system meaningful because that is what constrains its dynamics to be within the limits they express and as such explain how the parts of the system relate in order to be. In direct terms therefore, the basic reason why TA and FA remain widely employed (and the market wickedly impossible to "master" by them), is that those **disciplines** (and their variants in DFT, etc) reflect the limits of understanding generally available to market participants (with respect to the **dynamical structure** of markets).

Chaotic systems are linearly unstable but nonlinearly stable (the conundrum for traders)

In order to trade a chaotic system and be consistently successful, it must be obvious to the trader what the **local limits** are in a **global frame** that suggests the largest **immediate** objective of the market (each trade). Without this kind of knowledge (structure) in a linearly unstable system, it becomes a gamble to action reads. The trader is simply not sure of what is going on and everything soon begins to appear random. Because the trader observes sequences in fast **non-monotonic** evolution, the trader requires the specialized knowledge of chaotic dynamics and a setup that militates against the "confusion" arising from chaotic properties of the variable price to read an emergent fractal structure per period. Therefore, if a dynamical market system is **deterministic** but **generally unpredictable;** and if in addition it evolves by persistent **cyclic** trends (aperiodic cyclicality), and is also known to have a **fractal structure;** then it is possible to prescribe an **interpolative model** that is a **general model** of the market and that will exploit it **consistently.** This means that based on the knowledge of how such a market is dynamically ordered, it is possible to **read** and **trade** such a market with a consistency of result that demonstrably outperforms the market. As such, the **market model** can be shown, not just to be more effective than any predictive linear models of the market, but to be the correct general trading model of the market.

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Comparing the "predictivity" of linear methods and the chaotic nonlinear method

The **"predictability"** of any **chaotic** system depends on (1) how much error or uncertainty we are willing to tolerate in a given forecast or future estimate of the chaotic variable (2) how accurately we are able to measure a system's current state, and (3) **Lyapunov** time (a time scale reflecting the time from initial conditions to the point when a chaotic system becomes **unpredictable**). Therefore, these provide a basis by which to judge the **"predictivity"** of different approaches on a comparative scale. The claim here is that it is possible to improve current "predictability" of outcome from say 1/2 to say 1/20 in terms of 1 and 2 above, and as such, greatly clarify the empirical sense of Lyapunov time per interval in the case of 3. This model relies on applying the tenets of Fractal Geometry and Chaos theory to reading and trading markets, i.e. it directly employs real-time **Fractal Analysis** of markets.

Understanding the theoretical basis for the Multifractal Trading Model

A **fractal** is a never-ending pattern. **Fractals** are infinitely complex patterns that are self-similar processes across different scales. They are created (i.e. computer generated models of fractals) by repeating a **simple** process indefinitely in an ongoing feedback loop. Mathematically, any real system that describes the same kinds of functions is a fractal. We can say that **fractal geometry** is to **chaos theory** what geometry is to algebra in expressing the **mathematics** of chaos. A power of **Fractal Geometry** is the ability to model (explain) the explicit dynamics of chaotic systems. This allows two **equivalent** senses of deterministic chaos: (A) a system that "**appears**" to have "**random**" arrangement in space and or (B) "random" progression in time. This is extremely consequential since graphical concepts and insights tend to be much easier to grasp. Fractals are infinitely complex (that is to say **detailed**). This means fractal phenomena can be explained (modeled) to infinitesimal detail. **Fractal dimension** is the measure of such complexity - i.e. the ratio of the change in Scale to that in Detail. The important point that is made here is that all of this analytic power allows insights into complex dynamical systems in ways not possible before the science was formalized by Benoit Mandelbrot 41 years ago. **Multifractals** are a generalization of fractals not characterized by a **single** dimension. Rather, they express a **continuous** spectrum of **dimensions** reflecting complex dynamical forces in play. Multifractals are often found in experience. Coastlines, clouds, lightning, the human heart and electronically traded markets are examples

"Fractal geometry is not just a chapter of mathematics, but one that helps Everyman to see the same world differently." - Benoit Mandelbrot.

The Multifractal Trading Model

Market Microstructure

Regardless of how orders flow through (the different **dialects** of) an electronic trading system, in the end, there must be a **matching protocol** that simplifies the interface between **bids** and **offers**. For this model, the lowest **offers** frontend an array of sell orders to match an array of buy orders – with the highest **bids** at the front end. And this applies across variable **chunks** of orders reaching the market between session open and close (and combining with existing orders). In processing chunks, elements either side of the market (bids/offers) are spaced by point value (and **order size**) and therefore dynamically define **"demand** and **supply schedules".** So, these are vigorously populating ranges in market time that generate many changing variables and as a result, we have a myriad of sources per interval of not so obvious fluctuations defining the price curve in real-time (in addition to the more obvious one we describe further on). This state reflects something called **intermittency** (or aperiodic cyclicality - the signature sign of chaos). In other words, this algorithmically driven matching engine defines an intensely unstable scheme in dynamically processing latent **"demand** and **supply curves"** in market time. Importantly (and as we show next), the output of this process per interval, is simply **market clearing;** which information then feeds trader reaction over the same intervals.

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Clearly, not only can we visualize the movements implied, etc from this basic order matching scheme, but we can infer from this what a **pivot** is. A pivot then, is that **clearing price** point (order book level), that exhausts two matched (oppositely signed) arrays, where there are no further matches ahead in the current interval, or in such proximity, as to sustain an **initial direction**. We of course abstract from the fact that limit orders provide liquidity and market orders consume liquidity. But as long as a current range is actively populating with orders (i.e. orders are queued either side of the market), the range is not cleared and a pivot is not established to enable a reversal in such a range. Therefore, a pivot occurs, if the market exhausts oppositely signed arrays at a price point (order book level), with "**momentum"** (active queue) still on one side to reach a **"new"** range of oppositely signed orders. As such, **spot pivots** pervade the entire trading space including the shortest possible interval. This system does not exhibit stable equilibrium over any term, equilibrium is everywhere unstable.

Simplicity and Recursion

Traded markets function at least 24/5 all year round (and stock exchanges for significant periods of each weekday). During this time, all that goes on (per session) in terms of **microstructure** dynamics (barring one or two abstractions), is what we have described, regardless of the variety of; player types, order sizes, and investment horizons, intentions and influences, news and market events or what have you. As such, all that happens in the market all of the time is buying and selling. Therefore, the market at its most basic defines an incredibly **simple** form of existence: It admits all incoming buy/sell orders per interval, matches them and signs the flow. **Mr. Market** is **mechanical**. As such, the apparent complexity of traded markets comes from the **incessant** and **recursive** but even actions of market participants across different scales, and not from any other direct measures of their humanity. This is to say, a simple pattern repeated over and over again (i.e. indefinitely) on different scales within a fixed structure --> intimates a **fractal**.

Fractal primitives defined by the mathematics of Fractal geometry

1. So we deduce from ALL of this - that for price to "move" in market time, based on **order flow,** we need a three point structure for any aggregation of orders on any scale, large or small (the **initiator**).

2. That is to say, that there must be, for uprising price, two **buy** pivots and one **sell** pivot and for downfalling price, two **sell** pivots and one **buy** pivot to generate **directed** movement or **flow.** The Iterated Function Systems (IFS) formalism of fractal geometry would call each instance **(**a fractal **generator).**

3. As such, a **fractal primitive** (the fractal generator) in this model, simply consists of 3 consecutive range clearing price points (order book levels) in the same flow. These are price points in micro space (and therefore macro space) at each of which price reverses direction in sequence to define a price cycle. This underlying transactional framework repeating (iterated) indefinitely across **all** scales (the **rules of recursion**).

In effect, fractal primitives define as **volatility** shaped (self-affine) **linear inequalities** in market space (regardless of scale). Therefore, primitives are the best predictors of direction immediately following in the same transaction space or flow. As linear inequalities, there can be only 6 different definitions in any flow (combining in a diverse number of ways). In other words, fractal primitives affirm the (**exact**) extent to which predictivity anywhere exists in market space. Fractals allow (among other things) the trader to evaluate the state of the chaotic system per interval, and therefore across time scales. The stress here being seamlessness; from market **microstructure** to market **macrostructure,** and what is more, evidence of the feedback loop in the cyclical flow of markets.

"Fractals should be the default, the approximation, the framework." - Nassim Nicholas Taleb.

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From Micro to Macrostructure

Fractals Scale across Market Space

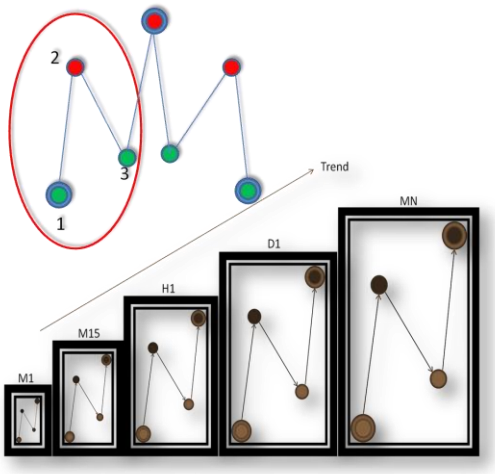

Figure 3: Fractals Scale across the entire Market Space

So we have that, market space is space where price fractals define aperiodic cycles across time in simple (but nested) 1,2,3 legs - from sub-minute time compression through MI – MN and in fact at **(> MN)**. Fractal behavior whether over very small or very large intervals is the same. Importantly, the smaller ones are nested in the bigger ones (i.e. Fractals nested within fractals or **equivalently** nested within phases of larger fractal cycles) all with the same form, and hence the deterministic (i.e. moves to only one other next state depending on the current state), dynamically scaling, **Multifractal structure** of markets.

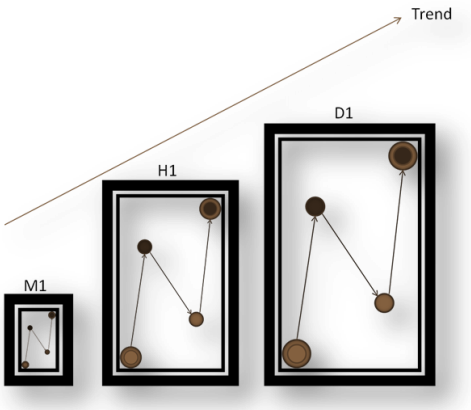

Figure 4: "Trending" Example

In the simplified, primitive driven "trending" example, each cell shown is a time frame, so we have a 1-minute frame, a 1-hour frame and a 1-day frame inscribed with 1.5 cycles or oscillations in each case. We are saying that the 1-minute frame contains at least one and a half cycles of sub-minute bars just to make its 1-2 leg **up** and at least one and a half cycles to make its 2-3 leg **down** and at least one and a half more to make up the last phase or half cycle shown. The same logic runs through the one hour frame and the one day frame in terms of their makeup in sequential flow (in practice the nested (iterated) **subcycle** count per feeder frame could be several more to the same effect but not less than 1.5 cycles per feeder frame). Of course, this implies (as in demonstrates) that the market is a **singularity** (a **point** in time). It means that the market is deterministic at the margin in dynamically scaling to a single form or **price point** across all intervals. As such there is no sense in which price does not move as a unit at all times and what is more, the marginal move is **NEVER** a **fair game**. Everything happens **(scales)** in (persistent) **sequence** following a fundamental order **(**iterated **hierarchy)** from left to right **(**until a **point)** and then **reverts** to origin **(finite loop equilibrium)** to resume **(once again)** left to right. This order does not change because the order expresses in the **rules of recursion** intrinsic to market mechanism.

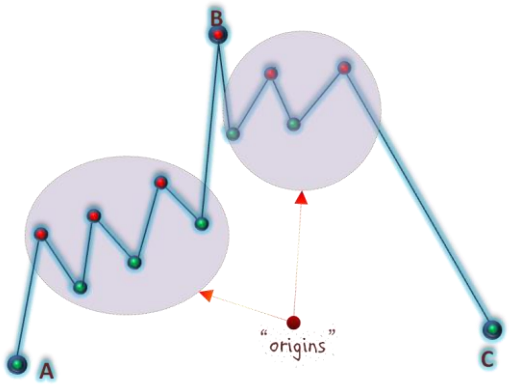

Figure 5: A Fractal Profile (single resolution)

So to cut a long story short, the fractal price structure is defined by nested (iterated), aperiodic (cyclical), price point displacement and the market can go up or down in range, over ALL time scales in downfalling or uprising runs.

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Never fine lines - rough lines - fractals spiral in and out of a nested (iterated) structure of finite loop equilibriums or "origin" (i.e. mean-reverting series), to resume scaling in direction or counter direction. Clearly, the setup is all the while driven by a nested (iterated) system of spot pivots defining fractal primitives across the entirety of market space. In fractal geometry, we hear mathematicians say that the **orbit** of some **seed,** s is attracted to **S**. S being a complete mapping of all s (the complete fractal form). Sometimes S is called a **strange attractor**. So yes, cycles, but not sine or cosine wave like. Instead, much rougher, aperiodic, non-monotonic, nested, infinitely scaled, i.e. fractal and therefore, illusory, when read outside of that structure, especially if focusing on a single resolution.

A Knowable Symmetry

As a result, (and this is key) – markets do not **"trend"** per **time frame** in any useful (tradable) sense. Markets **scale** as a **unit**, i.e. price spirals out of "origin" as a unit, going from left to right, across scales (up or down in range) to a **single point**. At such a point, price is back in "origin," fluctuating (i.e. spiraling out of "origin") to resume (up or down in range) as a unit, and from a single point - making time frames entirely arbitrary. This is **knowable symmetry;** as such price can be consistently timed with regard to the points in and out of any sequence of target "origins" in a given flow. Price movement is in no sense random. The **mathematics** of its dynamic is fixed (for all its incredible volatility). Markets are "**predictable**," i.e. within the limits of fractal behavior because electronically traded markets conform to a fundamental submission of Fractal Geometry; which is that, there is no **qualitative change** when the scale of a fractal object changes – a property known in mathematics as **"scale invariance."** The old school 50/50 risk/reward paradigm is sourced in the idea that the market is random (with the large baggage of emotions that entails for participants).

Trading by Chaos (Multifractal analysis and trading)

Trend is **NOT** your friend when the system is **chaotic**: consider the dynamic here where the points appear to be orbiting around and spiraling into a single point or singularity. Price dynamic is somewhat similar to these points on the Mandelbrot set attracted to a singularity. Measuring such dynamic for trend is almost meaningless and referencing a specific time frame for trade control pointless. Much of what we have described to this point is exactly reflective of this kind of dynamic.

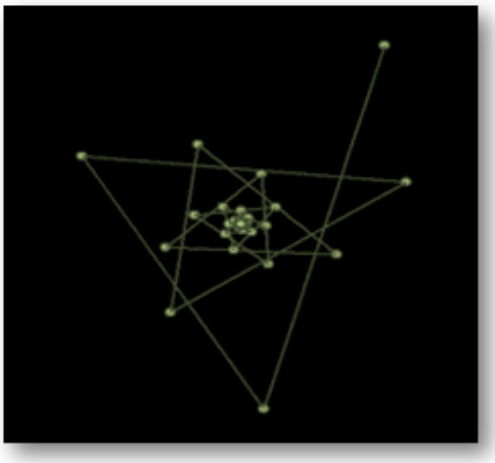

Figure 6: Points attracted to a Singularity

The trading objective, therefore, is to **isolate** and **time** tradable intervals (persistent series). We do this by dynamically accounting for the iterated sequencing, proportions, magnitudes and temporal periodicity of **Hurst types** as they combine (from left to right) and define a "discrete" flow. Fundamentally, this is a process that "unscrambles" time (i.e., defines the dynamic outside the notion of time frames). This means, we completely do away with trend analysis, focusing entirely on the fractal dynamics presenting within a calibrated fractal trading interface. Using the **IFS** sense of the market, we seek to follow the market as it hops from singularized point to **singularized point**. There is no place for second-guessing the market by recourse to events **exogenous** to the **scaling rules** under interpretation.

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Conclusion: "Unscramble time"

First, it is clear that the dynamical fractal structure defines market opportunities - in terms of timing and range. To be effective in such a space the trader must find a means to accurately read the local limits in a global frame that defines the largest immediate objective of the market. Among other nuances of number behavior, price is in **non-monotonic** evolution all of the time and the setup must account for the flow orientation implied by the emergent fractal structure. Second, there are at least three types of dynamics acknowledged by fractal geometry (Brownian series, Mean-reverting series and Persistent series), and it is the iterated proportions and sequencing of each type combining in real-time "discrete" flows (i.e. movements from singularized point to singularized point), that multifractal analysis must account for, to allow the trader time **tradable** intervals to near exactitude and to scale (Cf. Mandelbrot's clock time versus trading time). But third and even more profound is that - to read those Hurst types into validity, you "unscramble" time - my (Mis) Understanding of Mandelbrot.

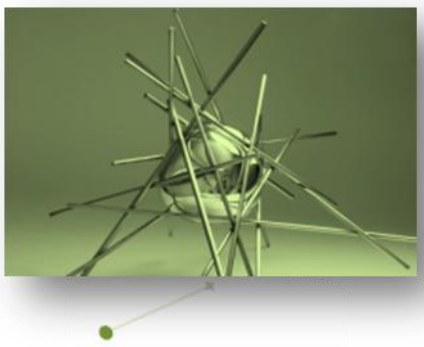

Figure 7: "Unscramble Time"

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