INDICATORS

Smoothing Techniques For More Accurate Signals

More sophisticated smoothing techniques can be used to determine market trend. Better trend recognition can lead to more accurate trading signals. Here's how.

by Tim Tillson

After studying his first stock chart, the novice technician is most likely to learn next about moving averages. This is a reasonable progression. First, moving averages are easy to understand. The simple moving average is just the average of a given number, which we will refer to as *n*, of previous closing prices, recalculated each day at the close. And second, technicians use moving averages because the moving average offers a smoother visual image of the market trend. In effect, the moving average removes the noise around the trend.

This concept of eliminating noise from the trend is similar to what engineers strive for in their application of digital filters. As R.W. Hamming observed:

> Digital filtering includes the process of smoothing, predicting, differentiating, integrating, separation of signals, and removal of noise from a signal. Thus many people who do such things are actually using digital filters without realizing that they are; being unacquainted with the theory, they neither understand what they have done nor the possibilities of what they might have done.

This quote applies to the vast majority of indicators in technical analysis. Moving averages, be they simple, weighted, or exponential, are *low-pass filters*; low-frequency components in the signal pass through with little attenuation or reduction, while high frequencies are severely reduced. Oscillator-type indicators, such as moving average convergence/divergence (MACD), momentum and relative strength index, are another type of digital filter referred to as a *differentiator*.

Early in the steps of calculating any of these indicators, the difference between today's price and some price a number of days ago or the difference between two moving averages are measured — hence the term. STOCKS & COMMODITIES Contributing Editor Tushar Chande has observed that many popular oscillators are highly correlated, which makes sense, because they are measuring the rate of change of the underlying time series.

We use moving averages (low-pass filters) in technical analysis to remove the random noise from a time series, to discern the underlying trend or to determine prices at which we will take action. A perfect moving average:

1. Would be smooth, not sensitive to random noise in the underlying time series. That is, its derivative would not spuriously alternate between positive and negative values.
2. Would not lag behind the time series from which it is computed. Lag, of course, produces late buy or sell signals that kill profits.

The only way a perfect moving average could be computed is to have knowledge of the future, and if we had *that*, we would buy one lottery ticket a week rather than trade! That aside, we can still improve on conventional simple, weighted, or exponential moving averages.

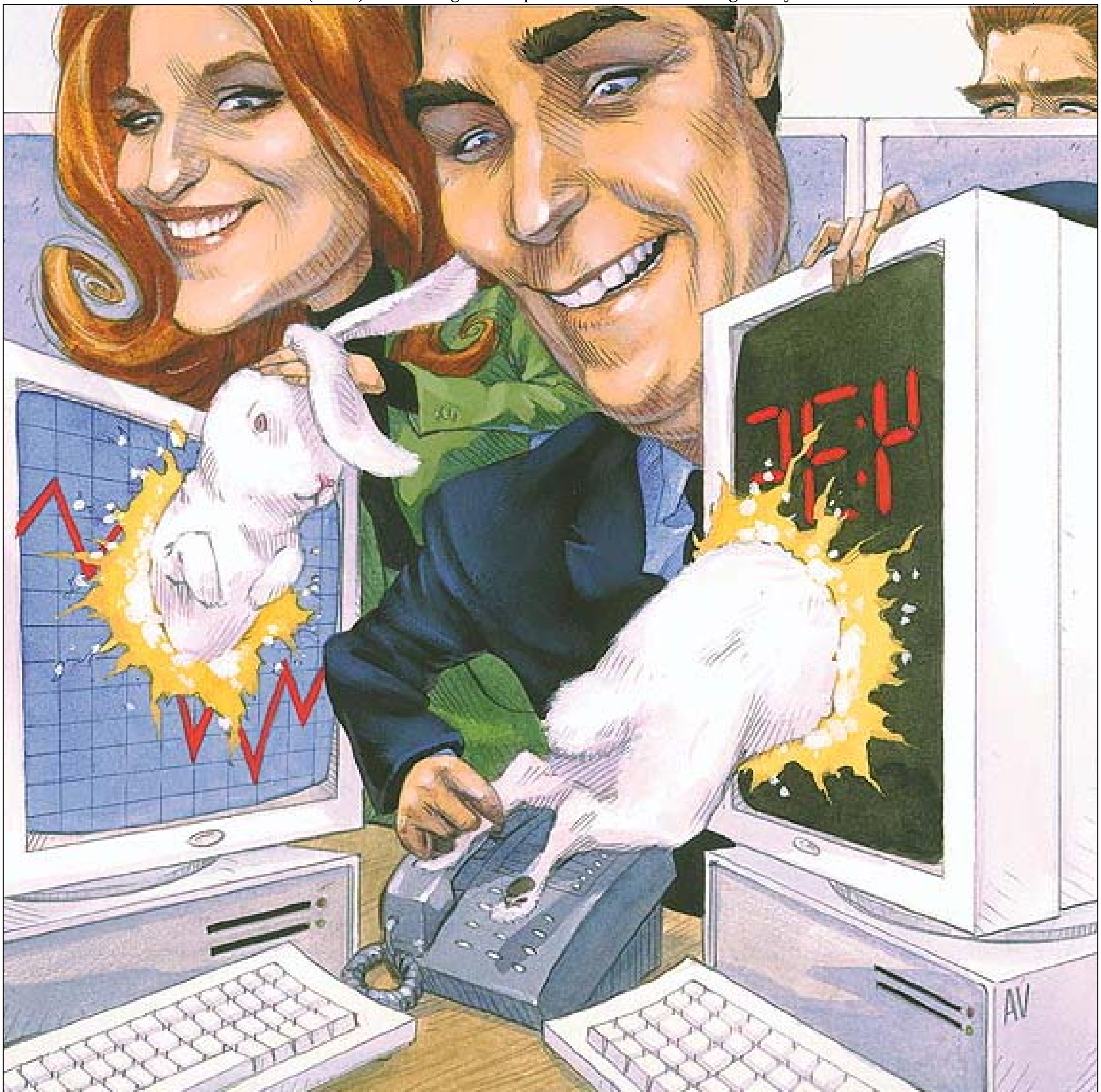
TWO BENCHMARKS

We will examine two benchmark moving averages based on linear regression analysis. In both cases, a linear regression line of length *n* is fitted to price data. The first benchmark moving average (MA) is called ILRS, which stands for *integral of linear regression slope*. In this moving average, the slope of a linear regression line is simply integrated as it is fitted in a moving window of length *n* across the data. The derivative of ILRS is the linear regression slope. ILRS is not the same as a simple moving average (SMA) of length *n*, which is actually the midpoint of the linear regression line as it moves across the data.

We can measure the lag of moving averages with respect to a linear trend by computing how they behave when the input is a line with unit slope. Both SMA(n) and ILRS(n) have lag of n/2, but ILRS is much smoother than SMA.

Our second benchmark is the *end point moving average* (EPMA). It is the endpoint of the linear regression line of length *n* as it is fitted across the data. EPMA hugs the data more closely than a simple or exponential moving average of the same length. The price we pay for this? EPMA is much noisier than ILRS, and it also overshoots the data when linear trends are present, as can be seen in Figure 1.

However, EPMA has a lag of zero with respect to linear input! This makes sense because a linear regression line will fit linear input perfectly, and the endpoint of the LR line will be on the input line.



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These two moving averages frame the tradeoffs that we are facing. On one extreme we have ILRS, which is very smooth but has considerable phase lag. At the other extreme, EPMA has zero phase lag, but it is too noisy and overshoots. We would like to construct a better moving average that is nearly as smooth as ILRS, but runs closer to where EPMA lies, without the overshoot.

A easy way to do this is to split the difference — that is, use (ILRS(n)+EPMA(n))/2. This will give us a moving average (call it IE/2) that runs in between the two, has phase lag of n/4 but still inherits considerable noise from EPMA. Can we build something that is comparable, but smoother? Figure 1 shows ILRS, EPMA and IE/2.

FILTER TECHNIQUES

Any student of filter theory would be able to tell you that the smoothness of a filter could be improved by running it through itself multiple times, at the cost of increasing phase lag.

There is a complementary technique (called *twicing* by J.W. Tukey, author of *Exploratory Data Analysis*) that can be used to improve phase lag. If *L* stands for the operation of running data through a low-pass filter, then twicing can be described by:

L' = L(time series) + L(time series - L(time series))

We add a moving average of the difference between the input and the moving average to the moving average. This is algebraically equivalent to:

2L-L(L)

This is the *double exponential moving average* (DEMA), popularized by STOCKS & COMMODITIES contributor Patrick Mulloy. DEMA has some phase lag (although it exponentially approaches zero) and is somewhat noisy, comparable to IE/2. We will use these two techniques to construct our better moving average after we explore the first one.

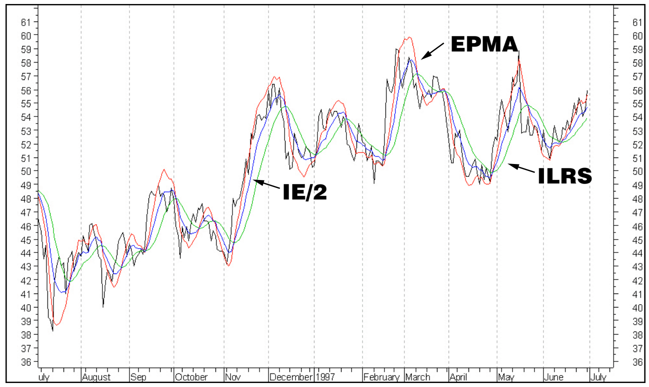
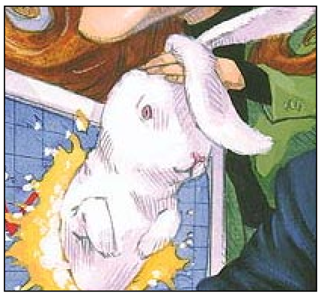


FIGURE 1: HEWLETT-PACKARD. The EPMA(15), IE/2(15) and ILRS(15) are red, blue and green, respectively in MetaStock. Note how the EPMA hugs the data more closely than a simple or exponential moving average of the same length. The price we pay for this is that it is much noisier than ILRS, and it also overshoots the data when linear trends are present.



FIXING OVERSHOOT

An *n*-day EMA has smoothing constant alpha=2/(n+1) and a lag of (n-1)/2. Thus, EMA(3) has lag 1, and EMA(11) has lag 5. Figure 2 shows that if I am willing to incur five days of lag, I get a smoother moving average if I run EMA(3) through itself five times than if I just take EMA(11) once. This suggests that if EPMA and DEMA have zero or low lag, why not run fast versions (for instance, DEMA(3)) through themselves a number of times to achieve a smooth result?

The problem with this approach is that multiple runs though these filters increase their tendency to overshoot† the data, giving an unusable result. This is because the amplitude response of DEMA and EPMA is greater than 1 at certain frequencies, giving a gain of much greater than 1 at these frequencies when run though themselves multiple times. Figure 3 shows DEMA(7) and EPMA(7) run through themselves three times. EPMA^3 has serious overshoot, and DEMA^3 similarly has problems.

The solution to the overshoot problem is to recall what we are doing with twicing:

METASTOCK IMPLEMENTATIONS

MetaStock 6.5 code for ILRS:

```
{input number of lookback periods, default is 11}
periods:=Input("Periods? ",2,63,11);

{determine how many points are in the time series}
size:=LastValue(Cum(1));

{determine the constant of integration by taking the simple moving average of the first periods points in the time series}
start:=LastValue(Ref(Mov(P,periods,S),periods-size));

{value is the integral of linear regression slope plus the constant of integration}
Cum(LinRegSlope(P,periods))+start;
```

If x stands for the action of running a time series through an EMA, *f* is our formula for generalized DEMA with the variable "a" standing for our volume factor:

$$f := (1 + a)x - ax^2$$

Running the filter though itself three times is equivalent to cubing *f*:

$$-a^3x^6+(3a^2+3a^3)x^5+(-6a^2-3a-3a^3)x^4+(1+3a+a^3+3a^2)x^3$$

Thus, the MetaStock 6.5 code for T3 is:

```
periods:=Input("Periods? ",1,63,5);
a:=Input("Hot? ",0,2,.7);
e1:=Mov(P,periods,E);
e2:=Mov(e1,periods,E);
e3:=Mov(e2,periods,E);
e4:=Mov(e3,periods,E);
e5:=Mov(e4,periods,E);
e6:=Mov(e5,periods,E);
c1:=-a*a*a;
c2:=3*a*a+3*a*a*a;
c3:=-6*a*a-3*a-3*a*a*a;
c4:=1+3*a+a*a*a+3*a*a;
c1*e6+c2*e5+c3*e4+c4*e3;
```

—*T.T.*

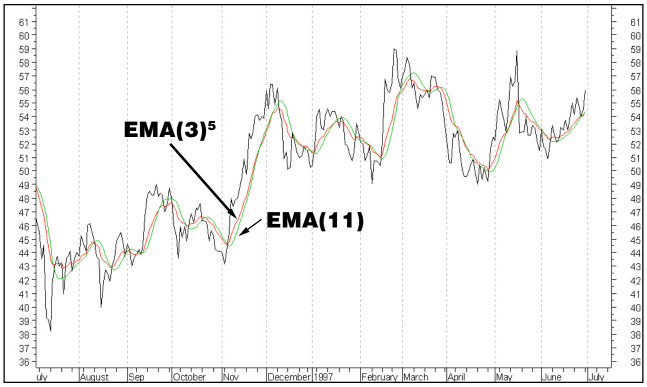


FIGURE 2: HEWLETT-PACKARD. In MetaStock, the EMA(11) is red versus the EMA(3)^5, which is green. EMA(11) and EMA(3)^5 both have five days of lag, but EMA(3)^5 is smoother.

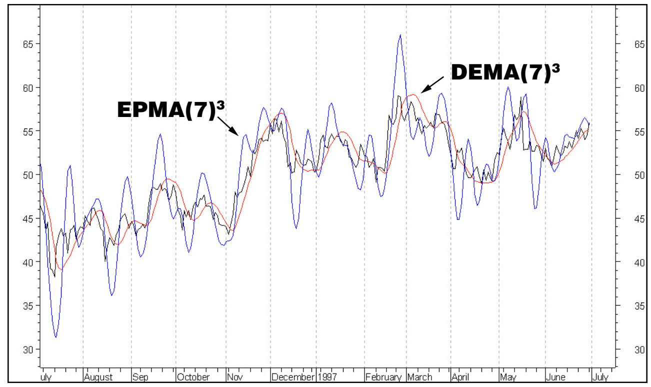


FIGURE 3: HEWLETT-PACKARD. In MetaStock, the DEMA(7)^3 is red and EPMA(7)^3 is blue. When DEMA(7) and EPMA(7) are run through themselves three times, DEMA^3 has serious overshoot, and EPMA^3 similarly has problems.

DEMA(n) = EMA(n) + EMA(time series - EMA(n))

The second term is adding, in effect, a smooth version of the derivative to the EMA to achieve DEMA. The derivative term determines how hot the moving average's response to linear trends will be. We need to simply turn down the volume to achieve our basic building block:

EMA(n) + EMA(time series - EMA(n)) * 0.7

This is algebraically the same as:

EMA(n) * 1.7-EMA(EMA(n)) * 0.7

I have chosen 0.7 as my volume factor. The general formula (which I refer to as "generalized DEMA") is:

GD(n,v) = EMA(n) * (1+v)-EMA(EMA(n)) * v

PERFECT MOVING AVERAGES AND OSCILLATORS

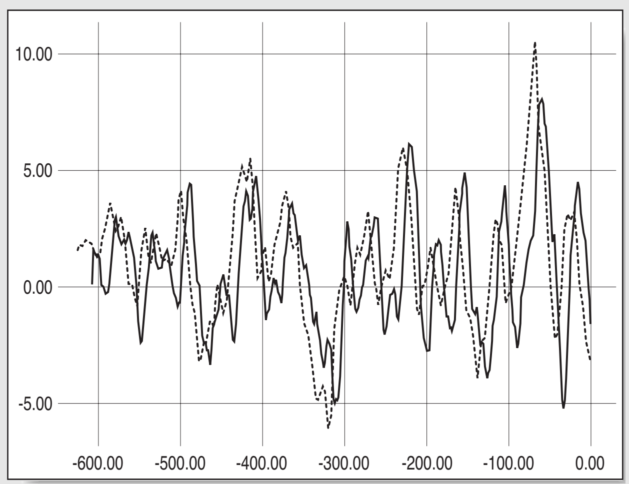
If we had knowledge of the future, perfect moving averages and oscillators could be constructed. Here's how it is done computationally:

- A perfect moving average can be constructed by adding an exponential moving average that moves *backward* in time to one that moves *forward* in time, then dividing by two. The phase lead of the backward EMA cancels the phase lag of the forward EMA, producing a moving average that is both smooth and in phase (but only for historical data, not the data on the right-hand side of the chart, where we want to trade).
- A perfect oscillator can be had by subtracting the forward EMA from the backward EMA, or by taking the derivative of the perfect moving average. These calculations are analogous to the two-sided derivative in calculus, which uses the future *f(x+h)* term in the definition:

$$\lim_{h \to 0} \frac{f(x+h)-f(x-h)}{2h}$$

In the real world, oscillators are constrained to be like the noisier left-sided derivative:

$$\lim_{h \to 0} \frac{f(x)-f(x-h)}{h}$$



SIDEBAR FIGURE 1: PERFECT(20). Tillson plotted the two-sided derivative as a dotted line, while the 20-day linear regression line slope is the black line.

The limitations of technical analysis become very apparent when one compares left-sided oscillators with perfect oscillators on historical data! Using about 600 days' worth of HWP daily closes, I plotted the two-sided derivative PERFECT(20) as a dotted line, while the 20-day linear regression line slope is the black line (sidebar Figure 1). PERFECT(20) has about 10 days of phase lead over LRS, about what one would expect. *—T.T.*

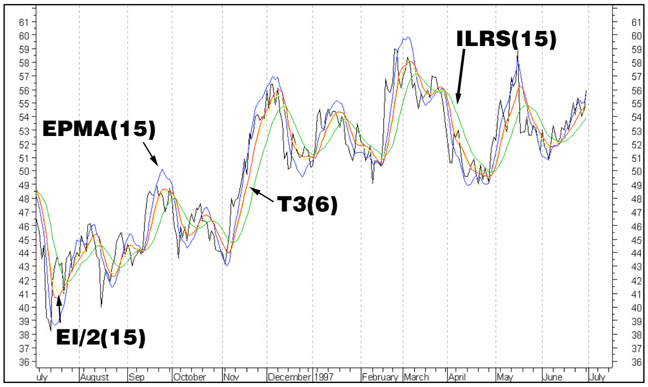


FIGURE 4: HEWLETT-PACKARD. The EPMA(15) is blue, the T3(6) is red, the IE/2(15) is yellow and the ILRS(15) is green in MetaStock.

where *v* ranges between zero and 1. When *v* = 0, GD is just an EMA, and when *v* = 1, GD is DEMA. In between, GD is a less aggressive version of DEMA. By using a value for *v* less than 1, we cure the multiple DEMA overshoot problem but at the cost of accepting some additional phase delay. Now we can run GD through itself multiple times to define a new, smoother moving average (T3) that does not overshoot the data:

$$T3(n) = GD(GD(GD(n)))$$

Figure 4 shows T3(6) plotted with EPMA(15), ILRS(15) and IE/2(15). T3 is very similar to IE/2 and DEMA, but smoother than both, which was our goal.

In filter theory terminology, T3 is a six-pole nonlinear Kalman† filter. Kalman filters are ones that use the error — in this case, (time series - EMA(n)) — to correct themselves. In the realm of technical analysis, these are called *adaptive moving averages*; they track the time series more aggressively when it is making large moves.

TRADING RESULTS

I used MetaStock 6.5 to compare five moving averages (SMA, ILRS, EMA, DEMA and T3) on the NASDAQ index (NDX) from July 19, 1993, to June 30, 1997, almost four years of data. I set the interest rate at 4% annualized, and a trading cost of 0.1% for entry and exit. This is realistic, since I can trade up to 1,000 shares through Fidelity Web Express for $14.95, and a typical trade might be 300 shares of a $50 stock.

MA	Optimal parameter	Best APR
ILRS	7	19.02
SMA	8	20.20
EMA	8	28.17
DEMA	7	28.81
T3	5 (volume = 0.7)	33.59

FIGURE 5: PARAMETERS AND PERFORMANCE. Here's a summary of the best results that each MA was able to achieve. Note that the optimal parameters did not vary widely.

The system used was very simple. A moving average was computed using each of the five above. A derivative was taken (one-period rate of change function). A long position was entered at bottoms and closed at tops, of the derivative. No shorts were taken. For example, the code for "enter long" for an EMA was:

```
res:=Mov(C,opt1,E);
d1:=ROC(res,1,points);
d1 > Ref(d1,-1) AND Ref(d1,-1) < Ref(d1,-2)
```

I let *opt1* vary from two to 10. This is a fast system, which executes a lot of trades. Figure 5 summarizes the best results that each MA was able to achieve; note that the optimal parameters did not vary widely.

No single set of backtesting results is conclusive. But these numbers confirm that T3 has merit — it not only looks good to the eye on the chart, but it can also be a powerful building block in other indicators and trading systems.

Tim Tillson is a software project manager at Hewlett-Packard, with degrees in mathematics and computer science. He has privately traded options and equities for 15 years.

RELATED READING

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Tukey, J.W. [1976]. *Exploratory Data Analysis*, Addison-Wesley: Reading, MA.

†See Traders' Glossary for definition

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